MAKING MEASUREMENTS

TIME:

The times that are required to work out the problems can be measured using a digital watch with a stopwatch mode or a watch with a second hand. When measuring the period of a ride that involves harmonic or circular motion, measure the time for several repetitions of the motion. This will give a better estimate of the period of the motion than just measuring one repetition.

DISTANCE:

Since you cannot interfere with the normal operation of the rides, you will not be able to directly measure heights, diameters, etc. All but a few of the distances can be measured remotely using one or another of the following methods. They will give you a reasonable estimate. Consistently use one basic unit of distance - meters or feet.

1. Pacing: Determine the length of your stride by walking at your normal rate over a measured distance. Divide the distance by the number of steps, giving you the average distance per step. Knowing this, you can pace off horizontal distances.

   I walk at a rate of _____ paces per ________....or....My pace = _______

2. Ride Structure: Distance estimates can be made by noting regularities in the structure of the ride. For example, tracks may have regularly spaced cross-members as shown in figure A. The distance **d** can be estimated, and by counting the number of cross members, distances along the track can be determined. This can be used for both vertical and horizontal distances.

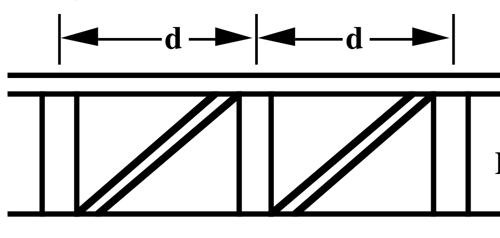


Figure A

3. Triangulation: For measuring height by triangulation, a horizontal accelerometer can be used. Suppose the height **h** of a ride must be determined. First the distance **L** is estimated by pacing it off (or some other suitable method). Sight along the accelerometer to the top of the ride and read the angle **θ**. Add in the height of your eye to get the total height.

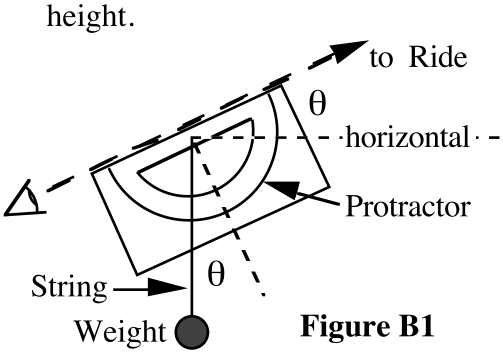


Figure B1

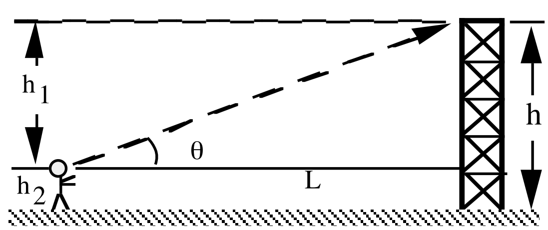


Figure B2

$\tan \theta = h_1 / L$, $h_1 = L \tan \theta$ h_2 = height of eye from ground
h = total height of ride = $h_1 + h_2$

4. A similar triangulation can be carried out where you cannot measure the distance to the base of the ride. Use the law of sines as illustrated in Figure C below:

Knowing θ_1, θ_2 and D, the height h can be calculated using the expression:

$$h = (D \sin \theta_1 \sin \theta_2) / \sin (\theta_2 - \theta_1)$$

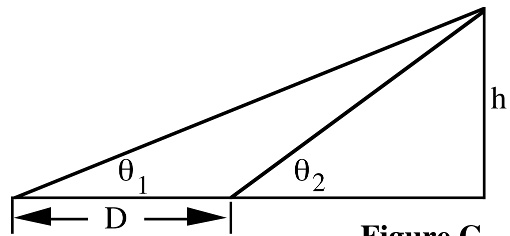


Figure C

SPEED:

The average speed of an object is simply distance divided by time. For circular motion, it is the circumference divided by time, if the speed is in fact constant.

$$v_{avg} = \Delta d / \Delta t = 2 \pi R / \Delta t \text{ (circular)}$$

To measure the instantaneous speed of a moving train, divide its length by the time it takes to pass a particular point on the track.

$$v_{inst} = \Delta d / \Delta t = \text{length of train / time to pass point}$$

In a situation where friction is ignored and the assumption is made that total mechanical energy is conserved, speed can be calculated using energy considerations:

$$GPE = KE$$
$$m g h = 1/2 m v^2$$
$$v^2 = 2 g h$$
$$v = \sqrt{2 g h}$$

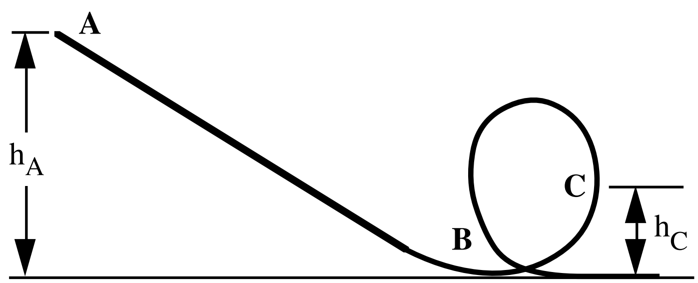


Figure D

Consider a more complex situation:

$$GPE_A + KE_A = GPE_C + KE_C$$

$$mgh_A + 1/2\, mv_A^2 = mgh_C + 1/2\, mv_C^2$$

Solving for v_C:

$$v_C = \sqrt{2 g (h_A - h_C) + v_A^2}$$

ACCELERATION:

Centripetal Acceleration

Calculations of acceleration in uniform circular motion are possible. Where **R** is the radius if the circle and **T** is the period of rotation, centripetal acceleration can be determined by the equations given below.

Centripetal Acceleration: $\mathbf{a_c = v^2 / R = 4 \pi^2 R / T^2}$

Direction of Acceleration

The net force that causes an object to accelerate is always in the same direction as the resulting acceleration. The direction of that acceleration, however, is often not in the same direction in which the object is moving. To interpret the physics of the rides using Newtonian concepts, you will need to determine the direction of the accelerations from the earth's (inertial) frame of reference. From this perspective, the following statements are true.

a) When an object traveling in a straight line speeds up, the direction of its acceleration is the same as its direction of motion.

b) When an object traveling in a straight line slows down, the direction of its acceleration is opposite its direction of motion.

c) When an object moves in a circle at a constant speed, the direction of its acceleration is toward the center of the circle.

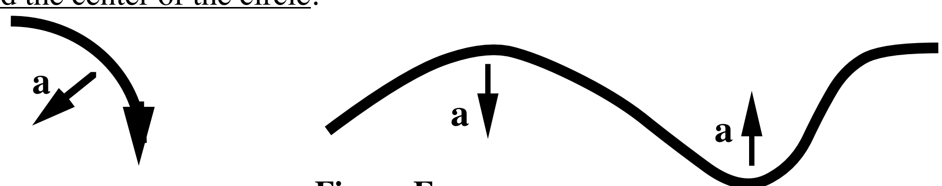


Figure E

d) When an object moves in a parabola (like those in a coaster ride), the direction of acceleration is along the axis of the parabola.

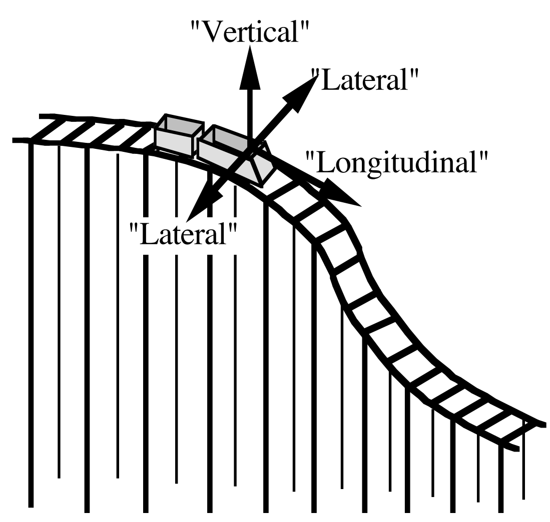
Naming the Directions

Vertical means perpendicular to the track

Longitudinal means in the direction of the train's motion

Lateral means to the side relative to the train's motion

Figure F



Accelerations on a Roller Coaster

Size of Accelerations

We use accelerometers that are calibrated in g's. The chart below contains examples of vertical accelerometer readings. A description of the sensation experienced is given.

Accelerometer Reading in g's	Sensation Experienced
3	3 times heavier than normal (maximum g's pulled by space shuttle astronauts)
2	Twice normal weight against the coaster seat
1	Normal weight against the coaster seat
0.5	Half of normal weight against the coaster seat
0	Weightlessness: No force of weight between the rider and the coaster seat
-0.5	Half the normal weight, but directed away from the coaster seat (as if the weight were measured on a bathroom scale mounted at rider's head!)

The Vertical Accelerometer

The vertical accelerometer gives an acceleration reading parallel to its long dimension. It is normally calibrated to read in "**g's**." A reading of **1 g** means an acceleration of 9.8 m/sec^2, the normal acceleration of gravity here on earth. Another way of stating this is to say that you are experiencing a force equivalent to your normal earth weight.

Note that there are three situations in which you may wish to use the vertical accelerometer: Head Upward, Head Downward, Sideways.

Head Upward:

This is when you are riding and your head is up, even though you may be going over a bump or going through a dip. An analysis of the forces gives us a net acceleration:

$$a_{net} = a_{reading} - 1\,g$$

Head Downward:

This is when you are at the top of a loop or a vertically circular ride and are upside down. Analyzing the forces here gives a net acceleration:

$$a_{net} = a_{reading} + 1\,g$$

Sideways:

This is when you are going around a horizontal curve, or you are measuring your starting or stopping acceleration. The accelerometer is held horizontal, and the reading is just equal to the net or centripetal acceleration.

$$a_{net} = a_{reading}$$

The Horizontal Accelerometer

The horizontal accelerometer is able to read accelerations that occur in a lateral or longitudinal direction. When going around a level corner with the horizontal accelerometer held level relative to the ground, pointed to the side, the angle of deflection gives a measure of the centripetal acceleration. The same technique would apply to longitudinal accelerations like the initial acceleration and final deceleration if the accelerometer is pointed forward in the direction of your motion.

From a force analysis it can be shown that the rate of acceleration is given by:

$$\mathbf{a = g \tan \theta}$$

Table of Tangents:

Angle	Tangent
0	0.00
5	0.09
10	0.18
15	0.27
20	0.36
25	0.47
30	0.58
35	0.70
40	0.84

Angle	Tangent
45	1.00
50	1.19
55	1.43
60	1.73
65	2.14
70	2.75
75	3.73
80	5.67
85	11.4

USEFUL RELATIONS

Distance, Velocity and Acceleration:

$v = \Delta d / \Delta t$ $a = \Delta v / \Delta t$

For Circular Motion...

$C = \pi d = 2 \pi R$ $v = C / T = 2 \pi R / T$

At the surface of the earth: $g = 9.8\ m/s^2 \approx 10\ m/s^2 = 32\ ft/s^2$

If acceleration is constant...

$d = (v_f - v_o)\ t / 2$ $d = v_o t + 1/2\ a t^2$

$v_f = v_o + a t$ $v_f^2 = v_o^2 + 2 a d$

Potential and Kinetic Energy:

Gravitational Potential Energy: $GPE = m g h$ h = height

Kinetic Energy: $KE = 1/2\ m v^2$

Force:

$F_{net} = m a$ Centripetal Force: $F_c = m v^2 / R = 4 \pi^2 m R / T^2$

Conversions:

88 ft/s = 60 mph 1.5 ft/s ≈ 1 mph 1 m/s ≈ 2 mph

1 ft/s ≈ 0.30 m/s 1 mph ≈ 1.60 km/h