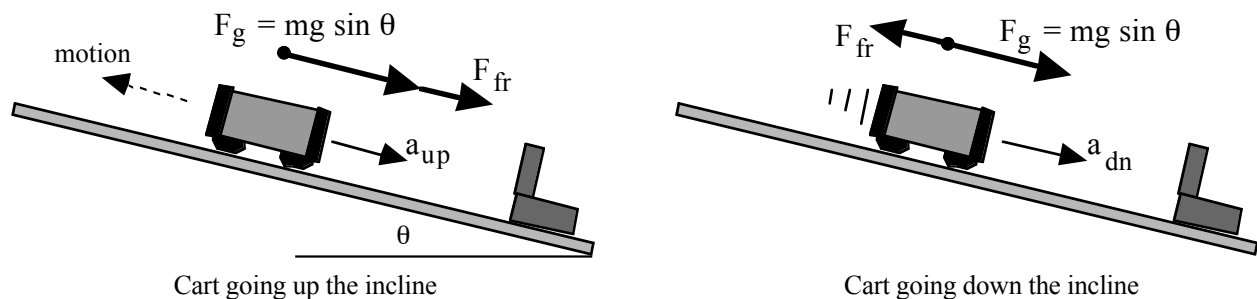


Cart on an Incline (Easy)

Theory:

As a laboratory cart goes up or down an inclined plane, it will be under the influence of two primary forces – gravity and friction. While it is comforting to consider only gravity, to fully understand the data one collects with a Motion Detector, you need to include friction in your work.

As the cart moves uphill (below left), the component of gravity and the friction force are both acting against the motion. This results in acceleration that we will call a_{up} . This is the acceleration that slows the cart while it is going uphill. From Newton's Second Law of Motion: $F_{net} = ma = (F_g + F_{fr}) = m a_{up}$.



As the cart moves downhill (above right), the component of gravity pulls it down while the friction force pulls uphill, working against its motion, resulting in a_{dn} . Using the Second Law: $F_{net} = ma = (F_g - F_{fr}) = m a_{dn}$. Note that absolute values of acceleration have been used in these equations.

Purpose:

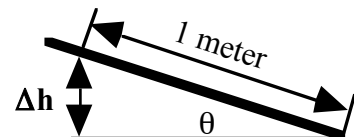
As you study the acceleration of a laboratory cart on an incline, you will determine the angle of the incline in two ways and calculate the effective friction force acting on the cart.

Equipment:

Motion Detector, TI graphing calculator, Inclined Plane (ramp or dynamics track), Laboratory Cart, Meter Stick, and either Go! Motion or LabPro and Motion Detector

Procedure:

1. Set up the inclined plane or dynamics track as shown above. Measure a length of 1 meter along the incline starting at the point where it touches the table. Measure the height from the table to the bottom of the incline at this point, Δh . Record this value in the Data Table.



Measure and record the mass of your cart, m . Limit Δh to 5 cm maximum.

2. Connect the Go! Motion to your calculator using a TI link cable, or connect the Motion Detector to your LabPro and use a link cable to connect the LabPro to the calculator. Place the motion detector near the bottom of the incline.

3. Launch **EasyData** under the [APPS] button. Press the button under $\overline{\text{File}}$. Choose **New**. The screen should indicate that a motion detector is attached and a time graph lasting 5 seconds should be indicated for Mode.
4. Set the cart approximately 20 cm from your motion detector (50 cm if using a “blue” Motion Detector). Practice pushing it uphill with just enough speed to rise near the top of the incline, but not going over the edge. Be sure to catch the cart before it crashes into your detector when it rolls back down.
5. Initiate data collection by pressing the button below $\overline{\text{Start}}$. When clicks are heard coming from the detector, push the cart up the incline as you practiced. A good set of data will show a region with a smooth curve on the distance graph.

Analysis:

1. Calculate the angle of the incline, θ_{trig} , from your Δh value. Note that $\Delta h/(1 \text{ meter})$ is the sine of angle θ . (Both numerator and denominator have to be in the same units!)
 2. Calculate the gravity force acting on the cart, **mg**.
 3. Press the button under $\overline{\text{Plots}}$ and choose the Velocity graph. On this graph, where is the cart moving uphill? Explain. Where is the graph moving downhill? Explain.
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4. Determine the acceleration manually while going uphill.
 - Press the right-arrow until the cursor is near the beginning of the region where the cart is moving uphill. Record the values for time (x_1) and velocity (y_1) in your data table.
 - Press the right-arrow until the cursor is near the end of the region where the cart is moving uphill. Record the values for time (x_2) and velocity (y_2) in your data table.
 - Calculate the acceleration (the slope of the velocity graph):

$$\mathbf{a} = \Delta \mathbf{v} / \Delta \mathbf{t} = \Delta \mathbf{y} / \Delta \mathbf{x} = (y_2 - y_1) / (x_2 - x_1) = (v_2 - v_1) / (t_2 - t_1)$$
 - This yields \mathbf{a}_{up} , the acceleration while going uphill, and you should expect it to be a negative value. Why?
 5. To determine the acceleration while going downhill, repeat the steps above, but for the region where the cart moves downhill. This gives \mathbf{a}_{dn} .
 6. You can use the procedure that follows to determine the accelerations using analysis tools built into EasyData.
 - First save your data into a file. Press the button under $\overline{\text{Main}}$ and choose **Save As**. Give your file a name like CART54. Press the button under $\overline{\text{OK}}$. Now press the button under [Graph] to return to the velocity graph.
 - When the Velocity graph is displayed, press the button under $\overline{\text{Anlyz}}$ and choose **Select Region**.
 - Use the right-arrow to move the blinking cursor to a position near the beginning of the uphill region then press the button under $\overline{\text{OK}}$. This sets the left-hand limit.
 - Use the left-arrow to move the blinking cursor to a position near the end of the uphill region then press the button under $\overline{\text{OK}}$.
 - The graph will be re-drawn with only this section shown.
 - Press the button under $\overline{\text{Anlyz}}$ then choose **Linear Fit**. The values for the slope and intercept will be shown. Write down the one which is the acceleration in your data table as \mathbf{a}_{up} .

- When you press the button under $\overline{\text{OK}}$ the data points are shown and the line representing the equation you were just given is drawn over them. If it is a good fit, move on to the next steps.
7. Press the button under $\overline{\text{Main}}$ to get to the main menu, then press the button under $\overline{\text{File}}$ and choose **Open**. Use the arrow keys to move to your file and press the button under $\overline{\text{OK}}$. Press the button under $\overline{\text{Graph}}$, then use $\overline{\text{Plots}}$ if necessary to get the velocity graph.
 - When the Velocity graph is displayed, press the button under $\overline{\text{Anlyz}}$ and choose **Select Region**.
 - Use the right-arrow to move the blinking cursor to a position near the beginning of the downhill region then press the button under $\overline{\text{OK}}$. This sets the left-hand limit.
 - Use the left-arrow to move the blinking cursor to a position near the end of the downhill region then press the button under $\overline{\text{OK}}$.
 - The graph will be re-drawn with only this section shown.
 - Press the button under $\overline{\text{Anlyz}}$ then choose **Linear Fit**. The values for the slope and intercept will be shown. Write down the one which is the acceleration in your data table as a_{dn} .
 - When you press the button under $\overline{\text{OK}}$ the data points are shown and the line representing the equation you were just given is drawn over them. If it is a good fit, move on to the next steps.
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8. If you add the two equations given in the theory, you arrive at the following relationship:

$$2 F_g = m (a_{up} + a_{dn}) = mg \sin \theta$$

where $F_g = mg \sin \theta$. Note that the equation assumes you are using absolute values of the accelerations, so drop any negative signs when doing this calculation. Now determine the angle of the incline, θ_{motion} , from your motion data.

9. Calculate the percentage difference between this value and the one you derived from using trigonometry and direct measurement.
10. If you subtract the two equations given in the theory, you arrive at the following relationship:

$$2 F_{fr} = m (a_{up} - a_{dn})$$

Determine the effective friction acting on the cart as it moved up and down the incline. If you divide the force of friction by $mg \cos \theta$, you obtain the coefficient of friction, μ .

Make a second determination of μ and compare it with the value you obtained for the moving cart.

Discussion Points:

1. As you examine the velocity vs. time graph, what do you notice about the graph line as it crosses the horizontal axis where $v=0$?
2. Does your percentage difference support the theory that was advanced at the beginning of this lab? What sources of error were there and how much would they likely have affected the results?
3. Do a quadratic curve fit for the position vs. time graph while the cart was going up the ramp. Compare the coefficients of this fit with the curve fit while the cart was going down the ramp. While you suspected the overall position vs. time graph was a parabola, it really is two parabolas joined at the point where the velocity is zero and the friction force changes direction.

4. Do you expect the friction force to have a larger effect on the two accelerations if the angle is larger or smaller? Why? You can test this.
5. Does the mass of the cart have an affect on the accelerations? You can test this.
6. If you change carts to one with more or less friction, will the results you found still hold? You can test this.
7. How does the amount of friction you obtained in Analysis step 5 compare to the value you would have calculated after measuring the effective friction along a level track? You can test this.
8. Relate the results of this lab to the problem of stopping an automobile either going uphill or downhill.

Data Table:
(manual fit)

Quantity	Symbol	Run 1	Run 2
1-meter height of incline	Δh		
Mass of cart	m		
Angle from trig	θ_{trig}		
Gravity Force	mg		
Uphill: v_1			
Uphill: t_1			
Uphill: v_2			
Uphill: t_2			
Uphill accel.	a_{up}		
Downhill: v_1			
Downhill: t_1			
Downhill: v_2			
Downhill: t_2			
Downhill accel.	a_{dn}		
Angle from motion	θ_{motion}		
Percent Diff.	%diff		
Gravity force	F_g		
Friction	F_{fr}		
Coefficient of Friction	μ		
Second determination	μ		
Percent Diff.	%diff		

Data Table 2
(Program fit)

Quantity	Symbol	Run 1	Run 2
1-meter height of incline	Δh		
Mass of cart	m		
Angle from trig	θ_{trig}		
Gravity Force	mg		
Uphill accel.	a_{up}		
Downhill accel.	a_{dn}		
Gravity force	F_g		
Angle from motion	θ_{motion}		
Percent Diff.	%diff		
Friction	F_{fr}		
Coefficient of Friction	μ		
Second determination	μ		
Percent Diff.	%diff		

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